Magnetic devices analysis by Face FEM coupled with standard reluctance network

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A Finite Element Method (FEM) mesh is converted to a reluctance network through an original magnetostatic formulation based on face shape functions. This meshed reluctance network is coupled with an standard one, characterizing a 0D system. Both approaches are fully-compatible and the hybridized problem can be solve with a single circuit solver. The approach is tested in 2D on a magnetic circuit with an air gap and compared to classical FEM nodal formulation.

***Index Terms*—Reluctance network, Finite Element Method (FEM), face shape functions, Face FEM**

I. INTRODUCTION

Optimizing electromagnetic devices can require a large amount of data that might be provided by numerical simulations. Many numerical methods are used to model electromagnetic devices, but the RNM (Reluctance Network Method) and FEM (Finite Element Method) are the most widely used for magnetostatic modeling.

The FEM is well known by its flexibility and generality, once the mathematical formulations are based on a mesh. Furthermore, it is noticeable the knowledge base available for this method, for instance [1]. However, it leads to an high number of degrees of freedom and so quite long computation times.

On the other hand, the RNM is one of the most primitive methods for magnetic modeling and its application is based on a reluctance network. This method has remained useful due to its coherent results obtained with low computational effort and low computational simulation time [2] and has been largely applied to model power transformers [3] [4]. This method is also largely applied to model rotating electrical machines [5][6][7] and transmission lines [8]. Nevertheless, it is important to notice that these applications are based on a reluctance network defined manually, that might imply an hard, long and non-general task.

In [9] is presented a methodology that couples nodal/edge FEM with external reluctances network. In this paper, we go a step forward by proposing a formulation fully-compatible with both numerical approaches and solved with a single 0D circuit solver.

Finally, the results of modeling a actuator with the classical nodal FEM and with the proposed methodology are compared.

II. MAGNETOSTATIC FACE FEM FORMULATION

The magnetostatic fields might be described by the Ampère's (1) and Gauss (2) laws and the constitutive relation for magnetic materials (3).

$$curl\,\mathbf{H} = \mathbf{J} \quad (1) \qquad div\,\mathbf{B}=0 \quad (2) \qquad \mathbf{B} = \mu\mathbf{H} \quad (3)$$

where the magnetic field $\mathbf{H}$, in A/m, is composed by the fields $\mathbf{H}_0$, due to an imposed current density source $\mathbf{J}_0$, and $\mathbf{H}_m$ that can be obtained from the gradient of the magnetic scalar potential V_r, as presented in (5).

$$\mathbf{H} = \mathbf{H_0} + \mathbf{H_m} \quad (4) \qquad \mathbf{H_m} = -\nabla V_r \quad (5)$$

Thus, applying (5) in (4) and integrating the resulting equation along a domain Ω, it is possible to obtain (6), for which $\mathbf{W}$ is the face interpolation function.

$$\int_\Omega \mathbf{W_i} \cdot \mathbf{H}\, d\Omega + \int_\Omega \mathbf{W_i} \cdot \nabla V_r\, d\Omega = \int_\Omega \mathbf{W_i} \cdot \mathbf{H_0}\, d\Omega \tag{6}$$

Since the magnetic induction $\mathbf{B}$, in T, is given by (7), the left side of (6) is rewritten as (8).

$$\mathbf{B} = \sum_{j=1}^{n_f} \mathbf{W_i} \Phi_i \tag{7}$$

$$\int_\Omega \mathbf{W_i} \cdot \mathbf{H} d\Omega = \sum_{j=1}^{n_f} \int_\Omega \Big(\upsilon \mathbf{W_i} \cdot \mathbf{W_j} d\Omega \Big) \Phi_j \tag{8}$$

where j and i are the faces index, nf is the number of faces, υ is the magnetic reluctivity, and Φ is the magnetic flux.

Applying the divergence theorem in the second term of (6) and evaluating it along two adjacent elements Ω_a and Ω_b, leads to (9). Taking into account that the normal component of the function $\mathbf{W}$ is constant along the face shared by Ω_a and Ω_b, its second term becomes null and it can be rewritten as (10).

$$\int_{\Omega_a+\Omega_b} \mathbf{W_i} \cdot \nabla V_r d\Omega = \oint_{\Gamma_a-\Gamma_b} V_r \mathbf{W_i} \cdot \mathbf{n} d\Gamma - \int_{\Omega_a+\Omega_b} V_r \left(\nabla \cdot \mathbf{W_i}\right) d\Omega \tag{9}$$

$$\int_{\Omega} \mathbf{W_i} \cdot \nabla V_r d\Omega = - \int_{\Omega_a + \Omega_b} V_r \left(\nabla \cdot \mathbf{W_i} \right) d\Omega \qquad (10)$$

This equation can be split in terms of Ω_a and Ω_b, where the flux direction is defined from Ω_a to Ω_b. Then, considering that $\nabla \cdot \mathbf{w_i}$ is the inverse of the element volume, it is possible to obtain (11).

$$\int_{\Omega} \mathbf{W_i} \cdot \nabla V_r d\Omega = V_{r\ a} - V_{r\ b} \qquad (11)$$

The third term of (6) is the magnetic field due to an imposed current density $\mathbf{J}_0$, which can be obtained using Biot-Savart law, for instance.

Finally, lets rewrite (6) as the following matrix system

$$[\Re]\,[\Phi] - [\Delta V_{Mean}] = [H_0] \qquad (12)$$

where $[\Re]$ is a reluctance matrix,$[\Phi]$ is the unknown flux matrix, $[\Delta V_{Mean}]$ is the magnetic potential jump between two reluctances and $[V_0]$ contains the fmm sources.

Besides the matrix system is stated, (2) is not solved yet. Nevertheless, the **B** free divergence is constrained solving (12) as a circuit system , where the Kirchhoff's current law is imposed. It suggests that the solution can be obtained by the use of a 0D circuit solver.

III. Model and Results

In order to compare the results obtained with the proposed methodology, a simple magnetic circuit with an air gap is analyzed using the well established nodal first order FEM, based on magnetic vector potential **A**. The magnetic induction distribution is presented in Fig. 1.

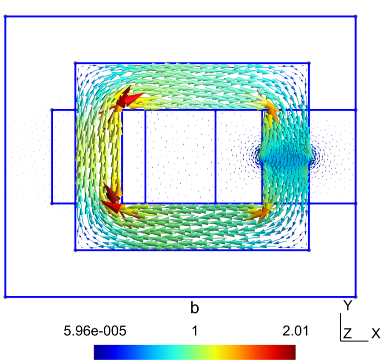

Fig. 1. Magnetic induction distribution obtained with classical FEM

Then, this model is reduced to a 0D model, i.e most of the magnetic circuit is modeled by an classical reluctance network and just the air gap region is meshed (i.e. the region where the equivalent reluctance network is not so easy to define), as presented in Fig. 2. Then the problem is solved by a circuit solver and magnetic induction is interpolated using (7), resulting in Fig. 3.

The maximum magnetic induction in the air gap obtained with the proposed methodology is 0.625 T and with the classical FEM is 0.606 T, that represents a difference of 3.13%.

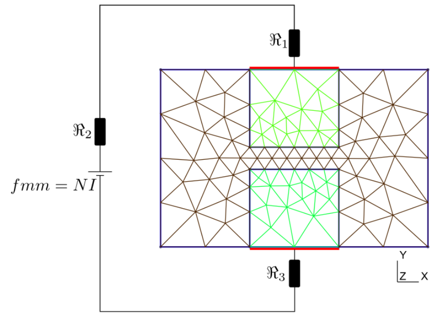

Fig. 2. Networks coupling

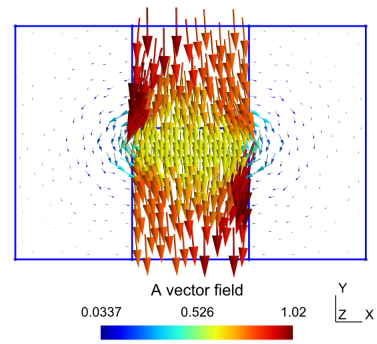

Fig. 3. Magnetic induction distribution along the air gap region, obtained with Face FEM.

IV. Conclusion

This paper has presented a methodology capable to convert a FEM mesh into a reluctance network, allowing its easy coupling with a classical network obtained analytically. The original model was reduced to a 0D system keeping coherent results.

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